

MATH 143: Week 10 - Monday's In-Class Exercises

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Lesson Plan: "Function Arithmetic"

Overview:

In this lesson our focus is function arithmetic; that is, taking the sum, difference, product, quotient, and composition of two functions using both symbolic and graphic representations. For this, we'll experiment with the arithmetic of various function families, including linear, quadratic, cubic, square root, and rational functions. In doing so, we'll perform calculations, sketch graphs, and make observations. At the end of the class, for the topics covered, the students will be given a short ungraded formative assessment that will request algebraic answers along with short written essay answers.

Important Note:

Although this is a dual class with both Math 108 and Math 143 students, this particular lesson plan focuses primarily on the Math 143 students (but much of the topics are the same). In this particular lesson, much of these topics are review for Math 143 and some new topics for Math 108.

Course Learning Outcomes (CLO):

Through in-class exercises and out-of-class exercises, successful students who pass this course will be able to:

1. **[CLO #1]** Apply and manipulate varied representations (i.e. symbolical, graphical, contextual, and tabular) of algebraic expressions, equations, inequalities, and functions to solve problems and puzzles in diverse contexts.
2. **[CLO #2]** Create algebraic functions with specific, important features such as: axis intercepts, degree, leading coefficient, asymptotes, rates of change, input and output, maximum and minimum, inverse, composition, domain and range.
3. **[CLO #3]** Analyze "real world" data with a regression analysis to estimate relationships among variables and make predictions. Such analyses will include scenario-based problems in disciplines such as business, science, engineering, and/or elementary education.
4. **[CLO #4]** Analyze a specific problem in a scenario and/or context in order to construct a written argument composed of complete English sentences, algebraic statements, and/or logical statements that reasons and explains:
 - i. Why a particular strategy, conclusion, and/or solution is appropriate.
 - ii. How the various components or ideas are related, connected, and/or organized.

Lesson Learning Objectives (LLO):

1. **[LLO #1]** Apply and manipulate various families of functions in symbolic and graphic representations with function arithmetic to solve problems, generate results, and make observations.
 - a. In alignment with: CLO #1
 - b. Fink: integration, application, foundational knowledge
2. **[LLO #2]** Analyze algebraic functions with specific, important features (such as degree, leading coefficient, and input-output ordered pairs) and use function arithmetic to construct the new resulting functions with additional new features to compare and contrast.
 - a. In alignment with: CLO #2
 - b. Fink: integration, application, foundational knowledge
3. **[LLO #3]** Analyze problem-specific scenarios with functions and function arithmetic in order to construct a written argument composed of complete English sentences that reasons and explains:
 - a. What are some similarities and differences between traditional number arithmetic and function arithmetic?
 - b. What is the dependent variable and independent variable of a function?
 - c. What is one benefit of using functions and function arithmetic?
 - d. What is the most challenging part(s) of learning about functions (i.e. muddiest point)?
 - e. Why functions and function arithmetic might be appropriate for "real world" situations?
 - i. In alignment with: CLO #4

- ii. Fink: integration, application, human dimension, caring

Prior Knowledge:

- Every student has had considerable exposure to families of linear, quadratic, cubic, square root, and rational equations and functions via in-class and out-of-class exercises, informal formative assessments, and one formative assessment; but learning about these function families hasn't required much knowledge of function arithmetic in this class thus far. Moreover, such topics haven't been emphasized or applied in much depth via in-class exercises to the extent that will be expected on Test 2 (in two and 1/2 weeks).
- Students are frequently adding, subtracting, multiplying, and dividing polynomial expressions on a daily basis, so today we'll build on these concepts and formally frame them in the context of functions. For example, we'll build on such prior knowledge by exemplifying that such polynomial expressions can be "organized" into functions for which arithmetic can be further applied. Also, the last three class periods, the students were exposed to rational functions, which is just one example of how two functions can be "connected" together arithmetically.

Situational Factors:

- There are 20 students in Math 143 along with 18 students in Math 108, so it will be a full class in a lecture classroom (i.e. not a computer lab classroom) designed to fit at most 42 students. The Math 108 students will sit on one side of the room, and the Math 143 will sit on the other side of the room so they can work in groups and so the instructor can target one audience at a time with my mini-lectures, feedback, and assessments.
- The single whiteboard is relatively small for this class size, so the whiteboard material will need to be clean and organized for students to see it.
- The class is from 6pm to 7:15pm, so it is an evening class when many of us are tired and have been working all day. (Note: as always, motivation and interactive exercises will be important to keep people active and working.)
- A relatively large percentage of the class is nontraditional students (in contrast to my previous experience of morning and early afternoon classes).

Lesson Procedure:

1. Introduction and Review

- **Exercise Type:** Mini-Lecture Semi-Active Activity
- **Learning Objective Alignment:** LLO #1; LLO #2; CLO #1; CLO #2
- **Estimated Time:** ~5 minutes
- **Routine:** Instructor will:
 - Recall the various function families and the concepts associated with traditional arithmetic.
 - Explain that traditional arithmetic can be extended to functions (i.e. recalling rational functions and such, polynomial division, etc.) via function arithmetic.
 - Show a short step-by-step example where students first attempt each step.

2. Basics of Function Arithmetic

- **Exercise Type:** Think-Pair-Share Interactive Activity and Assessment (Informal)
- **Learning Objective Alignment:** LLO #1; LLO #2; LLO #3; CLO #1; CLO #2; CLO #4
- **Estimated Time:** ~55 minutes
- **Routine:** Students will:
 - Form groups of 2-3 and begin collectively working on function arithmetic exercises, which will include (but not be limited to) the following topics:
 - The sum, difference, and product of two functions.
 - The quotient of two functions.
 - The composition of two functions.
 - Make connections between symbolic and graphic representations of functions, and compare and contrast various observed results associated with function arithmetic.
 - Use function arithmetic to construct new functions in accordance to certain featural specifications and a "real world" context.

- Experiment with function input-output by generating various ordered pairs in order to graph their creations and gain a visual sense.
- Have the opportunity to interact with the instructor and other students by asking questions and participating in various mini-lectures and think-pair-share exercises.
- (Hopefully) Have the opportunity (time permitting, and whiteboard-space permitting) to volunteer to go up to the whiteboard and share their creations with the class for a class-wide peer review assessment.
- Not submit these particular exercises for assessment, but instead will be assessed by the instructor via walk-by observation and inquiry.

3. Short Quiz and Reflection

- **Exercise Type:** Individual Quiz and Assessment (Informal)
- **Learning Objective Alignment:** LLO #1; LLO #2; LLO #3; CLO #1; CLO #2; CLO #4
- **Estimated Time:** ~15 minutes
- **Routine:** Students will individually:
 - Be quizzed on some of the function arithmetic topics covered with a single, multipart question, which is a condensed version of the main, previous think-pair-share activity that requests specific algebraic and numerical answers.
 - Reflect and recapitulate some aspects of the function arithmetic topics covered with a single, multipart, short essay response question, where they'll construct a brief, concise, written argument composed of English sentences that recapitulates some aspects of functions and function arithmetic (including muddiest points) as well consider some possible "real world" applications.
 - Submit their quiz and reflection work for an ungraded formative assessment.